

3.1 notes

VOCAB:

Percentile:

If an observation is in the p^{th} percentile, it means that...
 $p\%$ of the data is LESS THAN OR EQUAL TO the observation

So if you score in the 80th percentile on your test, this means that... you scored the same or better than 80% of your classmates

OR 80% of your classmates scored the same or less than you.

Examples:

1- I have the following data:

{2, 2, 4, 5, 6, 6, 6, 7, 8, 9, 9, 10, 12, 20}

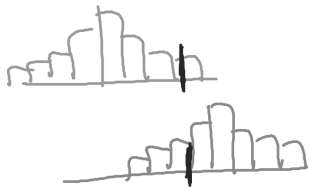
a) What percentile is the observation of "10"?

$$\frac{12}{14} = 85.71 \text{ percentile}$$

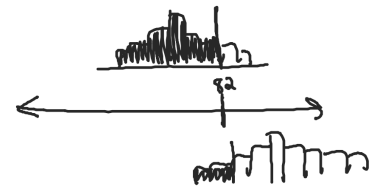
b) What percentile is the observation of "4"?

$$\frac{3}{14} = 21.43 \text{ percentile}$$

2- Sue got back her SAT scores and was told that she was in the 90th percentile for her school! She was very happy about this. However she was later told that she was also in the 40th percentile for students her age in the nation. She was not happy about this! How can this be true. What does this mean about the students at the school and students in the nation?



3- In two classes you took a test on Friday. On Monday you got both tests back and received an 82 on both. You then found out from the first class that your score for the first class was in the 64th percentile but your score in the second class was in the 46th percentile. How could this have happened?



Standardizing Observations

1) You are in a history class and a math class. You take your chapter 1 test in both classes on the same day, and get them both back a few days later. Your grades are as follows. Which class did you do better in?

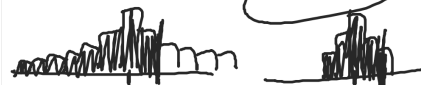
History: 81% Math: 75%

2) Same scenario. More info. Which class did you do better in?

mean: 76% mean: 70%
+5% +5%

3) Same scenario. More info. Which class did you do better in?

std. dev: 8% std. dev: 2.5%



$$z = \frac{x - \bar{x}}{s}$$

Standardizing Observations

Question: How can we compare one observation to another observation if they don't have the same parameters (mean and std. dev.)?

Answer: measure both observations in terms of their means & std. deviations- then we can compare them

math test vs. History test
76% 81%

STANDARDIZING:

- Gives the number of standard deviations an observation is away from its mean.

- Called a Z score



• $Z = \frac{\text{observation} - \text{mean}}{\text{std. deviation}} = \frac{X - \bar{X}}{s} = 1.2 \text{ std. deviations above average}$

- Gives direction: (+) = above mean (-) = below mean

- It has no units
- Does not change the shape of the distribution
- Changes the center by making the mean = 0
- Changes the spread by making the standard deviation = 1

EXAMPLE: Suppose for an IQ test the mean score is 100 with a standard deviation of 16.

- If a person scored a 120, what z-score would this be?

$\bar{X} = 100$
 $S = 16$
 $Z = \frac{120 - 100}{16} = 1.25 \text{ std. deviations above average}$

- If a person scored a 92, what z-score would this be?

$Z = \frac{92 - 100}{16} = -0.5 \text{ std. deviations below average}$

- What would a person's score be if the z-score was 1.8?

$1.8 = \frac{X - 100}{16}$
 $X = 128.8 \text{ units}$

- What would a person's score be if the z-score was -2.2?

$-2.2 = \frac{X - 100}{16}$
 $X = 64.8 \text{ units}$

EXAMPLE: Weight of Angus steers average 1152 lbs. with a standard deviation of 84 lbs.

What z-score would a steer have that:

1. Weighs 1000 lbs

$Z = \frac{1000 - 1152}{84} = -1.8095$

$\bar{X} = 1152$
 $S = 84$

2. Weighs 1250 lbs

$Z = \frac{1250 - 1152}{84} = 1.1667$



What weight would a steer be that has a z-score of:

3. $z = -1.5$

$-1.5 = \frac{X - 1152}{84}$

$X = 1026 \text{ lbs}$

4. $z = 1.4$

$1.4 = \frac{X - 1152}{84}$

$X = 1269.6 \text{ lbs}$

Example: The heights of 18-24 year old women are distributed with the following: mean = 64.5" std. dev. = 2.5"

We know a woman who is 69" tall. How does she compare to the rest of the women in her age group? Calculate her z score, and interpret what it means.

$Z = \frac{69 - 64.5}{2.5} = 1.8$

She is 1.8 std. deviations above avg. height

Example: We also know a man who is 71" tall. Who is taller relatively?

Men are known to be distributed with a mean of 67" and a std. deviation of 2.3 inches.

$Z = \frac{71 - 67}{2.3} = 1.739$

woman is taller, relatively.
b/c she is further above her avg than man.

Example: To be certain that you would be accepted to the college you want, you would need an SAT score of at least 1900. However you took the ACT. What score would you need to get? on ACT

For SAT mean = 1500 and $s = 250$

For ACT mean = 20.8 and $s = 4.8$

$Z_{\text{SAT}} = \frac{1900 - 1500}{250} = 1.6$

$Z_{\text{ACT}} = 1.6 = \frac{X - 20.8}{4.8}$

$X = 28.48 \text{ pts.}$

Complete the following from the book:

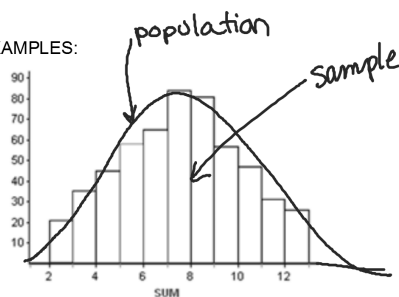
p. 105 #1 -- 6

#3-6

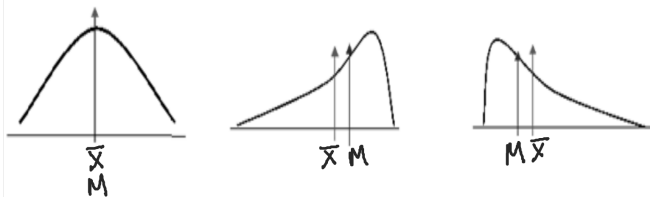
DENSITY CURVES

- A smooth curve that ... can be used to model the overall pattern/shape of a distribution (population)
- Can be any shape
- Total area ... of 100% (or 1) under the curve.

EXAMPLES:

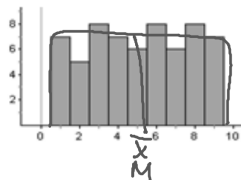
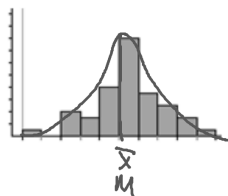
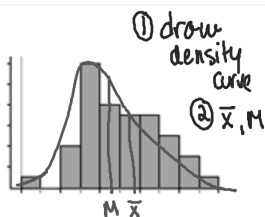
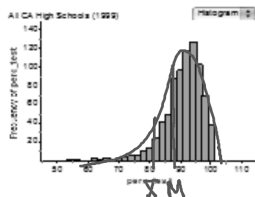


- MEDIAN of a density curve = the "equal-area" point
- MEAN of a density curve = the "balance" point



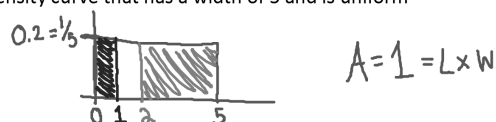
See page 110 in book!

EXAMPLES:



Example:

Draw a density curve that has a width of 5 and is uniform



a. What is the mean? The median?

$$\bar{x} = 2.5 \quad M = 2.5$$

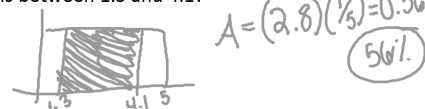
b. What % of data is below 1?

$$A = (1)\left(\frac{1}{5}\right) = 0.20 = 20\%$$

c. What percent of data is above 2?

$$A = (3)\left(\frac{1}{5}\right) = 60\%$$

d. What percent of data is between 1.3 and 4.1?



Complete the following from the book:

p. 110 # 8 -- 11 and p. 113 #13 --15

Complete Classwork 3.1

